BERKELEY MATH CIRCLE

The Math of Chemistry:

Moles & Molecules I

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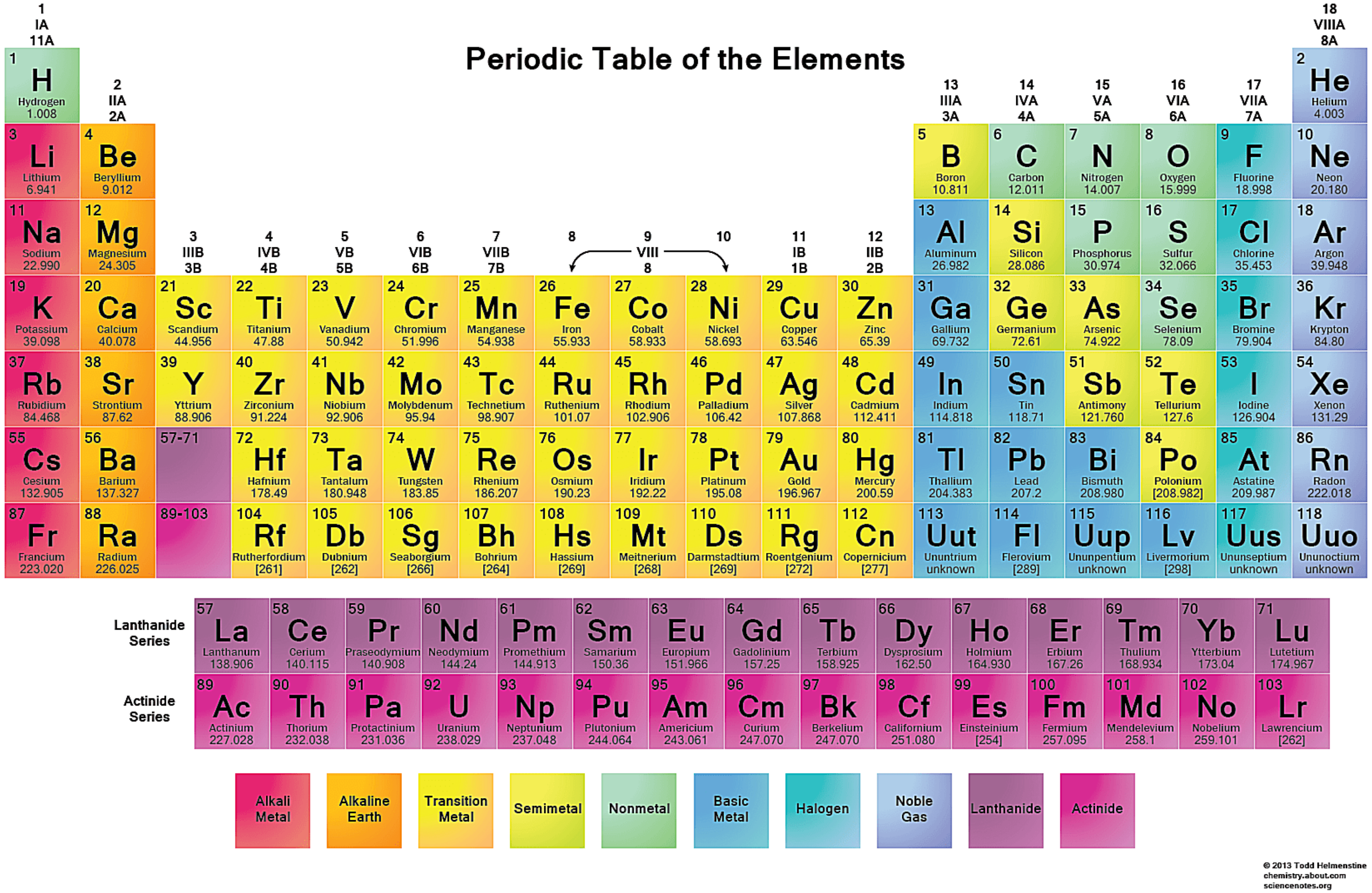
The Periodic Table

Elements are organized into the Periodic Table of Elements. They are organized into columns by their similarities in chemical properties:



FROM LAST TIME:

For each element, we can directly relate the amount of protons, electrons and neutrons that exist. But first, we need to learn some terms!

Symbol of Element

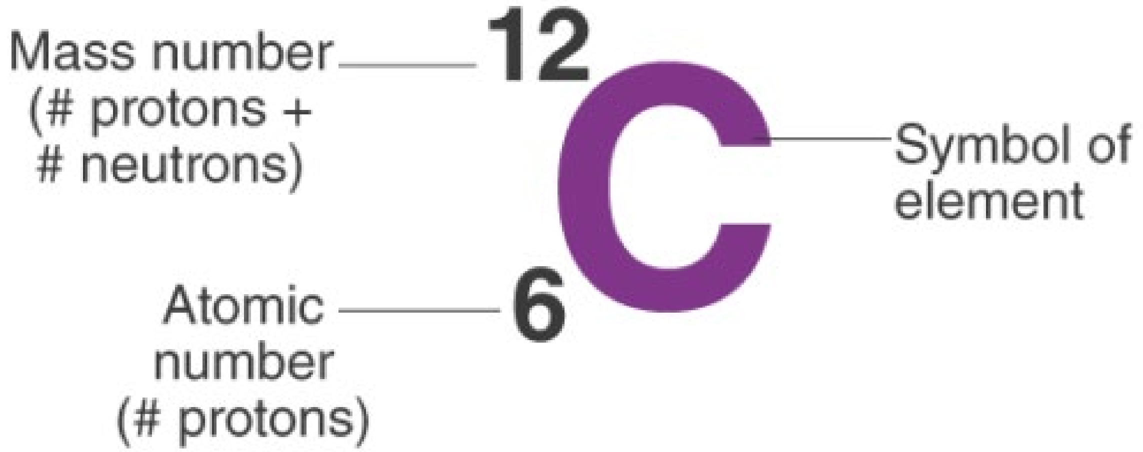
1 or 2 letter abbreviation for each element

Mass Number

- *Not always a whole number (more on this later!)*
- *#protons + #neutrons*

Atomic Number

#protons (defines the element!)



Thus,

# of protons = atomic number (defines the element!)
# of electrons = # of protons (if neutral)
# of neutrons = Mass Number - Atomic number

When we think of atoms, we can't "see" them, but we can measure them. We call this a **microscopic** view. But what about a **macroscopic** view? What things can we see, and then also measure? And how do we measure them?

For example:

If we measure our examples above, how do we compare them? By weight? By mass?

Since all of these elements exist as a collection of whole atoms, the only way to compare or measure them on **equal standing** is by measuring and counting **how many** of them exist. Their mass may be different, but the amount of them may be the same. In essence, we are counting how may particles of each element or molecule exist. Any guesses how many atoms in each of these?

Since we are dealing with very, very small objects, we need a way to count them in an easily digestible manner. Just like 1 dozen = 12 and 1 kilometer = 1000m, we need small, digestible values to represent INSANELY huge amounts!

We therefore welcome the concept of **Avogadro's Number** (NOT Avocado!)!

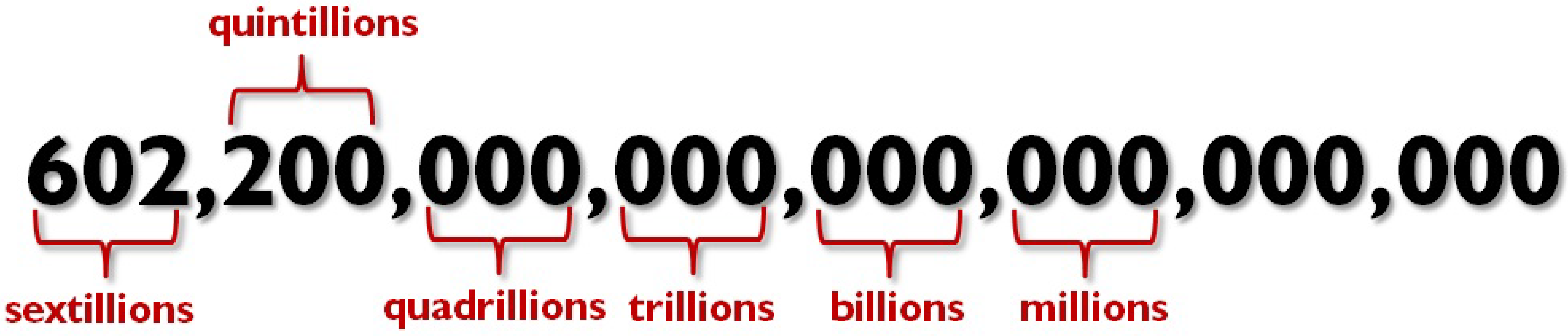
Avogadro $\neq$ **Avocado**

Avogadro's number is **6.022×10^{23} .** That's a HUGE amount!

How huge? Well . . . .

602,200,000,000,000,000,000,000

602.2 Sextillion!

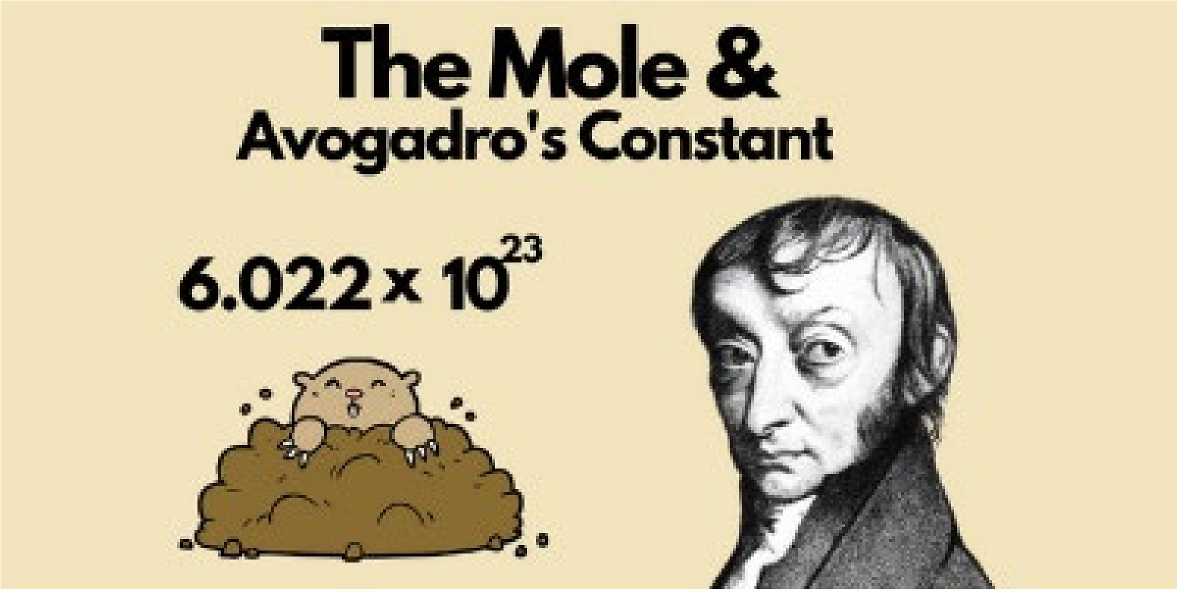


What then, does this HUGE number represent?

6.022×10^{23} = 1 **mole** of things (atoms, particles, molecules)

Again, just like:

1 dozen = 12
1 kilo = 1000
1 mole = 6.022×10^{23}



So, we've learned two new concepts:

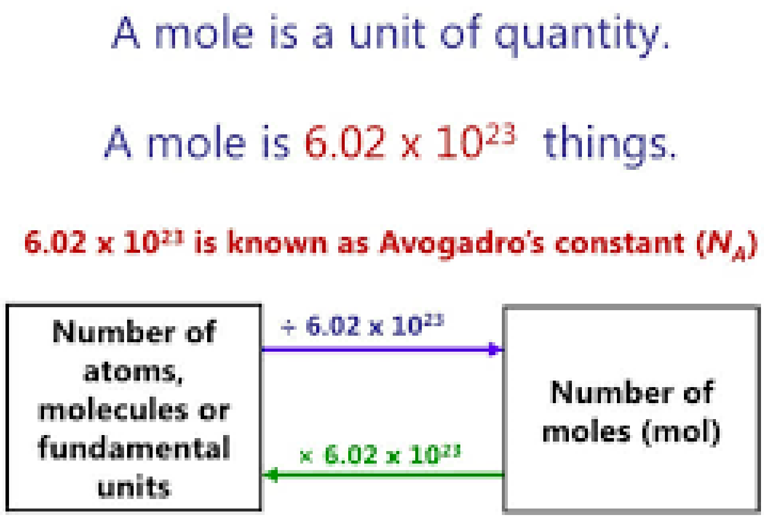
Avogadro's Number AND the **Mole**

How does all relate back to our microscopic and macroscopic view of things? From before:
Since all of these elements exist as a collection of whole atoms, the only way to compare or measure them on equal standing is by measuring and counting how many of them exist.

So:

1 mole of atoms = Avogadro's number of atoms

1 mole of atoms = 6.022 x 10^{23} atoms



Again, from before:
Their mass may be different, but the amount of them may be the same. In essence we, are counting how may particles of each element or molecule exist.

We will relate all of this to mass soon, but first, let's play with scientific notation, powers of ten, and numbers of things!

Math Fun!

To simplify, let's use 6×10^{23} = 1 mole of things for Avogadro's Number. You DO NOT need a calculator ;)

1) I have 2 moles of carbon. How many atoms of carbon do I have?

2) I have 0.5 moles of salt (NaCl). How many molecules of salt do I have?

3) I have 5 moles of water (H_2O). How many molecules of water do I have?

Math Fun!

To simplify, let's use $6 \times 10^{23} = 1$ mole of things for Avogadro's Number. You DO NOT need a calculator ;)

1) I have 2 moles of carbon. How many atoms of carbon do I have?

$$\frac{2 \text{ moles Carbon}}{1} \times \frac{6 \times 10^{23} \text{ atoms of Carbon}}{1 \text{ mole of Carbon}} = 2 \times (6 \times 10^{23}) = 12 \times 10^{23} = \mathbf{1.2 \times 10^{24} \text{ atoms of carbon}}$$

2) I have 0.5 moles of salt (NaCl). How many molecules of salt do I have?

$$\frac{0.5 \text{ moles NaCl}}{1} \times \frac{6 \times 10^{23} \text{ molecules of NaCl}}{1 \text{ mole of NaCl}} = 1/2 \times (6 \times 10^{23}) = \mathbf{3 \times 10^{23} \text{ molecules of NaCl}}$$

3) I have 5 moles of water (H_2O). How many molecules of water do I have?

$$\frac{5 \text{ moles } H_2O}{1} \times \frac{6 \times 10^{23} \text{ molecules of } H_2O}{1 \text{ mole of } H_2O} = 5 \times (6 \times 10^{23}) = 30 \times 10^{23} = \mathbf{3 \times 10^{24} \text{ molecules of } H_2O}$$

More Math Fun!

To simplify, let’s use 6×10^{23} = 1 mole of things for Avogadro’s Number. You DO NOT need a calculator ;)

1) I have 2.4×10^{28} atoms of Calcium (Ca). How many moles of Calcium do I have?

2) I have 3×10^{20} molecules of carbon dioxide (CO_2). How many moles of CO_2 do I have?

3) I have 1.8×10^{25} particles of helium gas (He). How many moles of helium do I have?

More Math Fun!

To simplify, let's use 6×10^{23} = 1 mole of things for Avogadro's Number. You DO NOT need a calculator ;)

1) I have 2.4×10^{28} atoms of Calcium (Ca). How many moles of Calcium do I have?

$$\frac{2.4 \times 10^{28} \text{ atoms of Calcium}}{1} \times \frac{1 \text{ mole of Calcium}}{6 \times 10^{23} \text{ atoms of Calcium}} = \frac{2.4 \times 10^{28}}{6 \times 10^{23}} = \frac{24 \times 10^{27}}{6 \times 10^{23}} = 4 \times 10^{4} = \mathbf{40{,}000 \text{ moles Ca}}$$

2) I have 3×10^{20} molecules of carbon dioxide (CO_2). How many moles of CO_2 do I have?

$$\frac{3 \times 10^{20} \text{ molecules } CO_2}{1} \times \frac{1 \text{ mole of } CO_2}{6 \times 10^{23} \text{ atoms of } CO_2} = \frac{3 \times 10^{20}}{6 \times 10^{23}} = \frac{1 \times 10^{20}}{2 \times 10^{23}} = 0.5 \times 10^{-3} = 5 \times 10^{-4} = \mathbf{0.0005 \text{ moles } CO_2}$$

3) I have 1.8×10^{25} particles of helium gas (He). How many moles of helium do I have?

$$\frac{1.8 \times 10^{25} \text{ particles He}}{1} \times \frac{1 \text{ mole of He}}{6 \times 10^{23} \text{ particles He}} = \frac{1.8 \times 10^{25}}{6 \times 10^{23}} = \frac{18 \times 10^{25}}{6 \times 10^{23}} = 3 \times 10^{2} = \mathbf{300 \text{ moles He}}$$

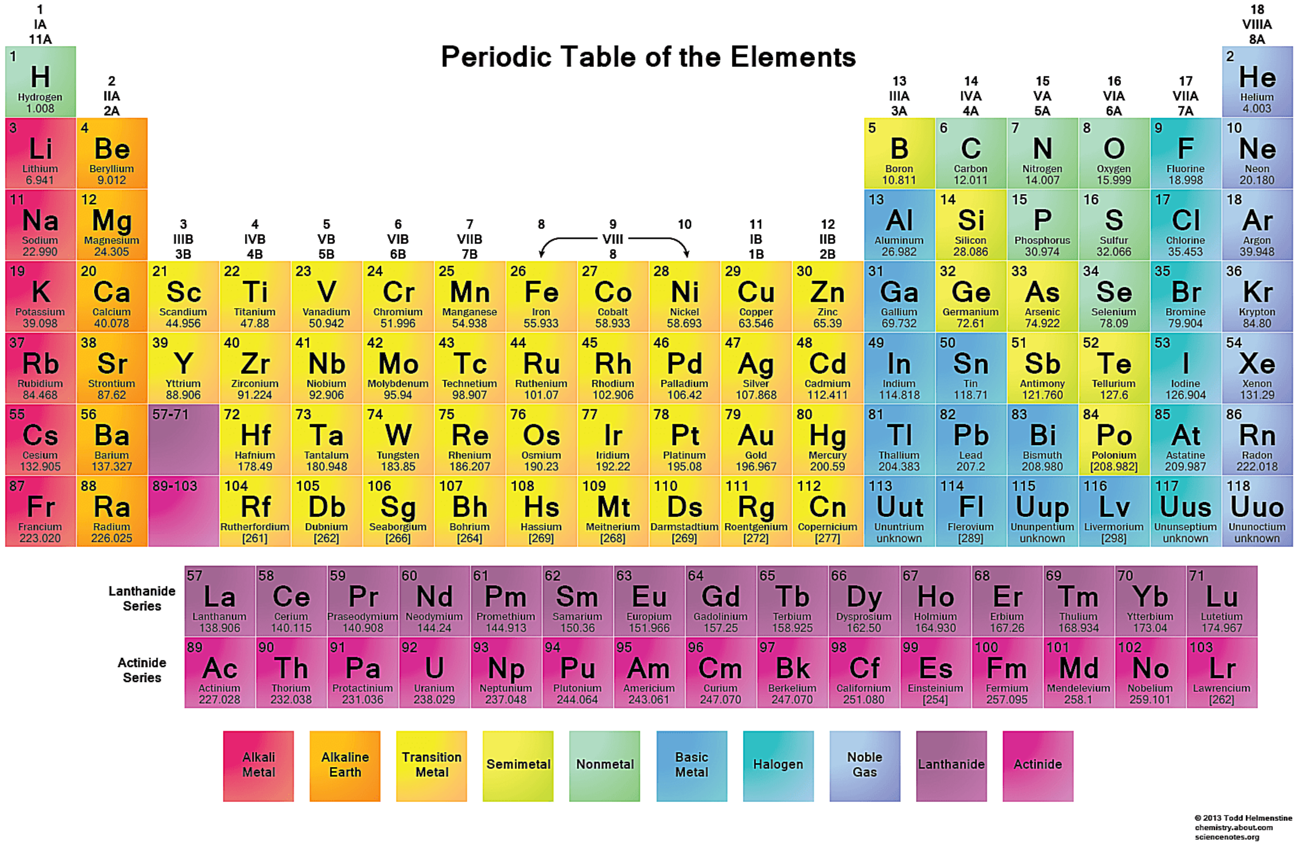
BUT, how does this relate to what we see and measure in day-to-day life! From before, we had the objects below. Can we say?

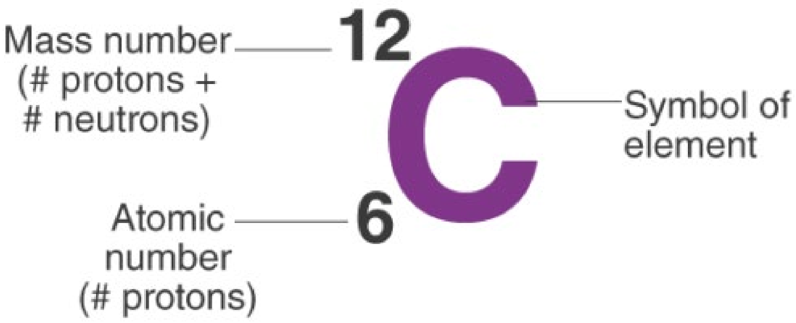
“I have 6.022×10^{23} atoms of carbon in my pencil”
“Please pass me 3.01×10^{23} particles of salt”
“May you fill my balloon with 1.04×10^{24} atoms of Helium?”

Of course not! **That’s ridiculous!**

We tend to measure things by their mass, and then relate them to how many of them exist (atoms, molecules, moles, etc.). So, we need to a way to do so . . . .

Our good friend, **The Periodic Table of Elements** is here to help us again! Without getting into all of the background, scientists have discovered that an Avogadro's amount of any element or atom = 1 mole of that element or atom, and is ALSO equal to that element or atom's atomic mass when measured in **grams.**





For Example:

6.022×10^{23} atoms of Lithium
1 mole of Lithium
6.941 grams of Lithium

6.022×10^{23} atoms of Aluminum
1 mole of Aluminum
26.982 grams of Aluminum

It’s also ADDITIVE! Table Salt = Sodium Chloride

+

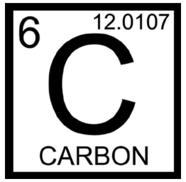
=

6.022×10^{23} molecules of NaCl
1 mole of NaCl
22.990 + 35.453 = 58.443 grams of NaCl

Even More Math Fun!

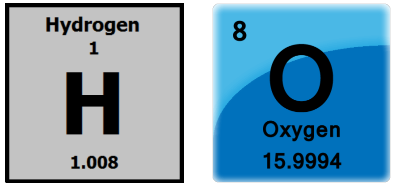
To simplify, let's use 6×10^{23} = 1 mole of things for Avogadro's Number. Also, round off mass numbers to their first decimal space. For example, Nitrogen's mass number is listed as 14.007, so use 14.0.

1) I have 2 moles of carbon. How many grams of carbon do I have?



2) I have 0.5 moles of salt (NaCl). How many grams of salt do I have?

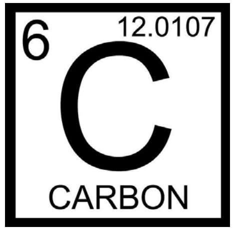
3) I have 5 moles of water (H_2O). How many grams of water do I have?



Even More Math Fun!

To simplify, let's use 6×10^{23} = 1 mole of things for Avogadro's Number. Also, round off mass numbers to their first decimal space. For example, Nitrogen's mass number is listed as 14.007, so use 14.0.

1) I have 2 moles of carbon. How many grams of carbon do I have?

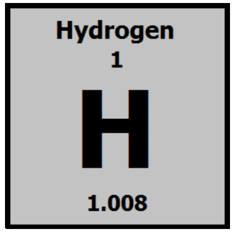


$$\frac{\text{2 moles of Carbon}}{1} \times \frac{\text{12 grams Carbon}}{\text{1 mole of Carbon}} = \textbf{24 grams of Carbon}$$

2) I have 0.5 moles of salt (NaCl). How many grams of salt do I have?

$$\frac{\text{0.5 moles of NaCl}}{1} \times \frac{\text{58.5 grams NaCl}}{\text{1 mole of NaCl}} = \textbf{29.3 grams of NaCl}$$

3) I have 5 moles of water (H_2O). How many grams of water do I have?



$$\frac{\text{5 moles of } H_2O}{1} \times \frac{\text{18 grams } H_2O}{\text{1 mole of } H_2O} = \textbf{90 grams of } \mathbf{H_2O}$$

More than Even More Math Fun!

To simplify, let's use 6×10^{23} = 1 mole of things for Avogadro's Number. Also, round off mass numbers to their first decimal space. For example, Nitrogen's mass number is listed as 14.007, so use 14.0.

1) I have 80.2 grams of Calcium (Ca). How many moles of Calcium do I have? How many atoms of Calcium do I have?

2) I have 132 grams carbon dioxide (CO_2). How many moles of CO_2 do I have? How many molecules of CO_2 do I have? *BONUS Question: How many atoms of Oxygen do I have?*

3) I have 30 grams of helium gas (He). How many moles of helium do I have? How many particles of He do I have?

The MOST Math Fun EVER!

To simplify, let's use 6×10^{23} = 1 mole of things for Avogadro's Number. Also, round off mass numbers to their first decimal space. For example, Nitrogen's mass number is listed as 14.007, so use 14.0.

1) Pick your favorite Element from the Periodic Table. Write down it's element abbreviation and mass number.
2) Square your age (use whole numbers only).
3) Your age squared is how many grams of your favorite element you have.
4) Calculate how many moles of your element you have.
5) Calculate how many atoms of your element you have.

Bonus Problem

Someone in class asked about what some of these items may look like in real life. The most familiar example is water (H_2O). If you poured a glass of water, how many water molecules are in the glass, i.e., how many molecules of water are you drinking?

Let's assume 1 cup of water:

1 cup = 8 fluid ounces = 237 mL (you can look up this conversion factor)
We need water's density to match up with we learned last week, which is 1g/mL.

So, 1 standard cup of water of contains 237 grams of water. We therefore calculate:

You can do similar thought problems with salt, or any other pure (or mostly pure) substance in your house.

What Have We Learned?

We relate quantities in Chemistry but how many PHYSICAL items exist. This is why knowing Avogadro's number and the concept of a mole is so important! If we are trying to calculate specifically what is happening, we MUST know how many things are interacting with how many other things, and we do so via moles. Therefore, the relationship below is KEY to understanding chemical interactions:

# of Particles←→ Moles ←→ Grams

Let’s Expand This!

Let’s use our NEW understanding to understand something we probably ALL have done: the famous at-home volcano experiment!

Does anyone know what is happening in that experiment?

Let's Expand This!

The basics are that we mix vinegar with baking soda and BOOM! But what's happening on a chemical level?

Vinegar contains Acetic Acid (CH_3COOH). This is found in liquid form.
Baking Soda contains sodium bicarbonate ($NaHCO_3$). This is found in solid form.

So,
$CH_3COOH + NaHCO_3 \rightarrow Na^+ + CH_3COO^- + H_2CO_3$

And then
$H_2CO_3 \rightarrow H_2O + CO_2$

Carbon Dioxide is a gas which then escapes into the air. This is what causes the "lava" movement in the experiment!

Relating back to Earlier . . .

$CH_3COOH + NaHCO_3 \rightarrow Na^+ + CH_3COO^- + H_2CO_3 \rightarrow H_2O + CO_2$

One can relate how much acetic acid and sodium carbonate will react (assuming a full reaction) by using our previous relationship and expanding on it . . . .

Previously we had . . .

# of Particles←→ Moles ←→ Grams

And we can use this for BOTH the acetic acid and sodium carbonate.

BUT, how do we relate them to each other?

The Mole Bridge!

of Particles←→ Moles ←→ Grams

of Particles←→ Moles ←→ Grams

The Mole Bridge!

The mole bridge is part of a concept in Chemistry called Stoichiometry.

Stoichiometry = the relationship between the relative quantities of substances taking part in a reaction

Back to volcanoes then . . . .

Volcano Problem

$\mathbf{CH_3COOH + NaHCO_3} \rightarrow Na^+ + CH_3COO^- + H_2CO_3 \rightarrow H_2O + CO_2$

If I used 100 grams of Acetic Acid, how many grams of Sodium Bicarbonate will I need to measure out in order to react all of the 100 grams of Acetic Acid?

# of Particles ←→ Moles ←→ Grams

⬍

# of Particles ←→ Moles ←→ Grams

Volcano Problem

CH_3COOH → 60g/1 mole

$NaHCO_3$ → 84g/1 mole

100 grams of CH_3COOH x 1/60 = 5/3 moles CH_3COOH →
5/3 moles $NaHCO_3$ x 84g/mole = **140g $NaHCO_3$**

of Particles ←→ Moles ←→ Grams

of Particles ←→ Moles ←→ Grams